May 16, 1878.

Sir JOSEPH HOOKER, K.C.S.I., in the Chair.

The Presents received were laid on the table, and thanks ordered for them.

The following Papers were read:—

I. "Experimental Researches on the Electric Discharge with the Chloride of Silver Battery. Part II. The Discharge in Exhausted Tubes." By WARREN DE LA RUE, M.A., D.C.L., F.R.S., and HUGO W. MULLER, Ph.D., F.R.S. Received April 10, 1878.

(Abstract.)

We cannot flatter ourselves that we have done more during our three and a half years' work than contribute a few facts towards the data necessary for the solution of the problem of the cause of stratification produced by the electric discharge in vacuum tubes. We refrain for the present from suggesting any hypothesis to account for this beautiful phenomenon, in the hope of being able to confirm, experimentally, certain views which we entertain concerning it. The paper we have now the honour of laying before the Society consists mainly of a record of the various appearances presented by the discharge in residual gases at various pressures and with various currents.

Throughout our labours we have felt strongly the necessity for obtaining measurements of the physical conditions (nature and pressure of the gas, strength of the current, difference of potentials between the electrodes, size of the tubes, forms of electrodes) which accompany the various phenomena observed during our experiments.

In order that others may be in a position to theorize as well as ourselves, we have given a more detailed account than we otherwise should have done of the phenomena observed, which, for the same reason, we have illustrated by copious diagrams and many copies of photographs.

For example, in diagram, fig. 43 are shown some of the very great variety of phases in rarefied hydrogen, observed with tube 129, 32 inches long, 1·6 inch diameter; the terminals being a straight wire and a ring about 1 inch in diameter, both aluminium. We quote a few observations by way of illustration:—

Pressure 16 millims., 21,053 **M** (millionths of an atmosphere), 8,040 cells. The ring being positive, curiously formed luminous entities shot at intervals from it, remained stationary for a time, and

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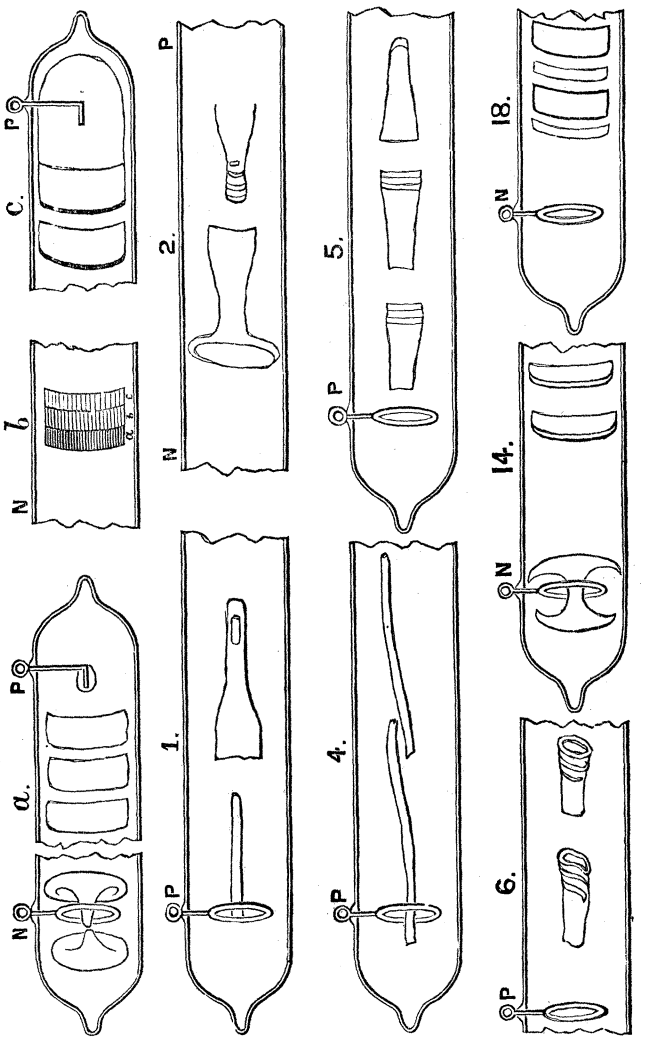
then disappeared, to be replaced by others; the flow, as seen in a rotating mirror, was towards the negative. The phenomena are depicted in 1, fig. 1.

Fig. 1.



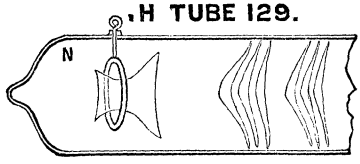
Pressure 12·2 millims., 16,053 M, 8,040 cells. The luminosities as in 5, fig. 1. The flow of the luminosities was shown by the rotating mirror to be towards the negative. There was much heat developed

in the neighbourhood of the luminosities, a little heat at the positive terminal, the negative being quite cool.

Pressure 10·8 millims., 14,211 **M**. Luminosities as in 6, fig. 1, which reminded one of a fish's mouth, especially as they opened and closed continually; they extended along 9 inches of the tube.

At a pressure of 1 millim., 1,316 **M**, the most beautiful phase of all was produced as shown in fig. 2, in some of its chief features, the current used was that of 2,160 powder cells, current 0·011520 Weber. The strata grouped themselves in threes and reached to within 6 inches of the negative ring; when 200,000 ohms resistance was introduced, C. 0·005658 **W**, the dark space extended to 9 inches. Only a portion of the luminosity about the negative ring is shown in the diagram; besides this, the ring was surrounded with a cylindrical nebulosity of about a $\frac{1}{4}$ inch in diameter, and a glow filled up the end of the tube.

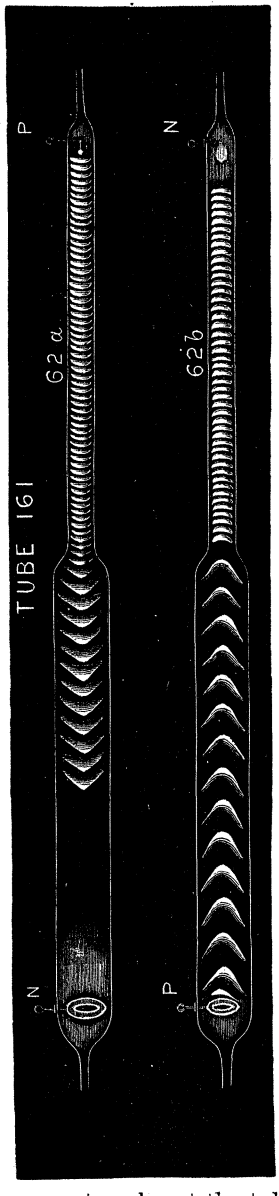
Fig. 2.



Tube 161, *Hydrogen.*

The difference of the strata in tubes of different diameter at the same pressure and with the same current is very clearly brought out in tube 161, fig. 3, composed of two portions, one being 18 inches long and 1·65 inch internal diameter, the other 17·5 inches long and 0·975 inch diameter, the ratio of the sectional areas being 2·864 to 1. The terminal in the small tube is a point, in the large one a ring. With 4,800 cells, the point (small tube) positive, C. 0·02825 **W**, there were produced in the small tube 62 disk-shaped strata, and in the large tube twelve cup-shaped strata occupying half of the length of the large tube; beyond these the discharge was dark. With the point negative C. 0·02451, there were produced in the small tube 54 disks, and in the large tube thirteen cup-shaped, completely filling it. The number of strata does not therefore appear to be in the inverse ratio of the areas. The strata in the small tube were blue, but at times, with a larger current, carmine, as in the capillary part of a spectrum analysis tube, the strata in the large tube being much fainter and pink. 62*a* and 62*b*, fig. 3, copied from photographs obtained, the former in 15 seconds, the latter in 10 seconds, show respectively the appearances at another phase, when the small tube was positive or negative respectively.

Fig 3.



We have found it necessary to exhaust the tubes ourselves, in order to be certain of the conditions as to pressure and purity of the gas (which we examined spectroscopically at various degrees of exhaus-

tion). By operating in this way we have been able to reproduce over and over again phases which are more or less transient in all tubes; indeed, we encountered great difficulties when we first commenced our labours with tubes supplied by various makers, for after a very short time they completely and permanently changed so as no longer to present the splendid stratifications witnessed on a first trial.

All the tubes which we exhausted were provided with two glass stop-cocks, one of which was connected with the gas generator, the other to the system of pumps. The rinsing of the apparatus was first effected many times with a high pressure water *trompe* (head of water 106 feet, producing a vacuum of 12 millims.); the tube was then exhausted by a mercurial pump (Alvergniat's make), and finally by the Sprengel. In our later experiments the measurements of low pressures were made by means of the M'Leod gauge, which indicated pressures down to 0·00005 millim.

By absorbing carbonic acid with potash, and, more especially, hydrogen with spongy palladium, a vacuum was obtained so perfect (pressure 0·000055 millim., 0·066 M.) that the potential of 11,000 was insufficient to cause a discharge.

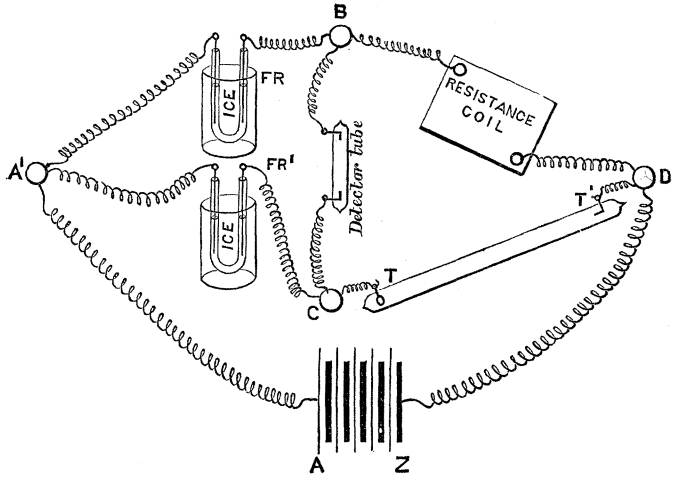
We have adopted several expedients for ascertaining the resistance of the tubes. In the first instance we placed the tube in one of the arms of a Wheatstone bridge, but the oscillations of the current were so great that it was necessary to substitute a vacuum tell-tale tube for the galvanometer; this became illuminated whenever the currents in the two branches were unequal. In spite of this device it was impossible to balance *exactly* the resistance of a tube by a metallic resistance. It was found that the resistance of a vacuum tube varied greatly as soon as the current passed and usually diminished, but when once the current was interrupted it was necessary to increase considerably the balancing resistance in the symmetrical branch of the Wheatstone bridge, in order to reproduce an illumination. After standing at rest for some time the tube took up its original condition.

In fig. 4 A Z is the battery, the A terminal of which is connected at A', in the bridge arrangement, with two equal fluid resistance tubes, FR and FR', of 420,000 ohms, placed in vessels containing ice, to keep them at a constant temperature; an adjustable coil resistance is inserted between B and D; the tube T T', to be tested, is placed between D and C, the Z terminal of the battery being connected to D. When the resistance is greater or less than that of the tube to be tested there is an illumination in the detector tube between B and C; but when a current passes in T T', balanced by a proper adjustment of the coil resistance, then the glow in the detector ceases.

Better results were obtained either by reproducing the deflections of a galvanometer, observed during the course of the experiments, with metallic resistance substituted for the tube; or by measuring with an

electrometer, the differences of potential between the electrodes of the tube, and between the ends of any convenient known resistance in the same circuit.

Fig. 4.



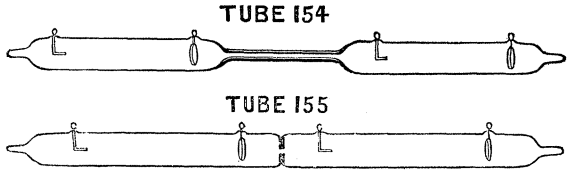
We were induced to extend the investigation by means of the electrometer, to the determination of the rate of fall of the potential throughout the length of a vacuum tube. For this purpose we had several tubes constructed with aluminium rings fixed to platinum wires inserted in the glass at nearly equal intervals. It was found that the fall of potential for equal spaces had a maximum value in the neighbourhood of the negative electrode; in the neighbourhood of the positive, though considerably less than near the negative, it was greater than in the middle of the tube, where for equal intervals the fall of potential was sensibly constant.

The most important result of these measurements was the establishment of the *constancy* of the difference of potential between the electrodes of a given tube and gas, at a given pressure, for all variations in the value of the current, even when these were as great as 1 to 150. This conclusion was arrived at subsequently to the observation of the majority of the phenomena recorded in the paper. Accordingly, it must be understood that what has been termed "resistance of a tube" indicates that the difference of potential between its terminals was the same as that which would have existed between the ends of a metallic wire of the resistance given if substituted for the tube in the particular circuit.

We have, moreover, by experimenting with such tubes as 154 and 155, fig. 5, wherein there is a constricture of the same bore, but of different lengths, established the fact that the constricture greatly increases the

resistance of the tube, but that this increase depends more upon the smallness of the diameter of the constricture than upon its length.

Fig. 5.



In the course of our experiments we have arrived at the following facts:—

1. *The discharge in a vacuum tube does not differ essentially from that in air and other gases at ordinary atmospheric pressures; it cannot be considered as a current in the ordinary acceptation of the term, but must be of the nature of a disruptive discharge, the molecules of the gas acting as carriers of electrification. The gases in all probability receive two impulses in opposite directions, that from the negative being the more continuous of the two.* Metal is frequently carried from the terminals and is deposited on the inside of the tube, so as to leave a permanent record of the spaces between the strata.*

2. *As the exhaustion proceeds the potential necessary to cause a current to pass diminishes up to a certain point, whence it again increases, and the strata thicken and diminish in number, until a point is reached at which, notwithstanding the high electromotive force available, no discharge through the residual gas can be detected.*

3. *All strata have their origin at the positive pole.* Thus, in a given tube, with a certain gas, there is produced at a certain pressure, in the first instance, only one luminosity which forms on the positive terminal, then, as the exhaustion is gradually carried further it detaches itself, moving towards the negative, and being followed by other luminosities, which gradually increase in number up to a certain point.

4. *With the same potential the phenomena vary irregularly with the amount of current.* Sometimes, as the current is increased, the number of strata in certain tubes increases, and as it is diminished their number decreases; but with other tubes the number of strata frequently increases with a diminution of current.

5. *A change of current frequently produces an entire change in the colour of the strata.* For example, in a hydrogen tube from a cobalt blue to a pink. It also changes the spectrum of the strata; moreover, the spectra of the illuminated terminals and the strata differ.

6. *If the discharge is irregular and the strata indistinct, an alteration*

* "Phil. Trans.," Part I, vol. clxix, pp. 90 and 118.

of the amount of current makes the strata distinct and steady. Most frequently a point of steadiness, sufficient for photographing, is produced by the careful introduction of external resistance; subsequently the introduction of more resistance produces a new phase of unsteadiness, and still more resistance another phase of steady and distinct stratification.

7. *The greatest heat is in the vicinity of the strata.* This can be best observed when the tube contains either only one stratum, or a small number separated by a broad interval. There is reason to believe that even in the dark discharge there may be strata; for we have found a development of heat in the middle of a tube, in which there was no illumination except on the terminals.

8. *Even when the strata are to all appearance perfectly steady, a pulsation can be detected in the current; but it is not proved that the strata depend upon intermittence.*

9. *There is no current from a battery through a tube divided by a glass division into two chambers, and the tube can only be illuminated by alternating charges.*

10. *In the same tube and with the same gas, a very great variety of phenomena can be produced by varying the pressure and the current. The luminosities and strata, in their various forms, can be reproduced in the same tube, or in others having similar dimensions.*

11. *At the same pressure and with the same current, the diameter of the tube affects the character and closeness of the stratification.*

II. "Note on Legendre's Coefficients." By I. Todhunter, F.R.S. Received April 16, 1878.

In the "Proceedings of the Royal Society," vol. xxvii, pp. 63–71, Professor Adams has given, by an inductive process, the development of the product of any two of Legendre's Coefficients in a series of the Coefficients; from this is immediately deduced the value of the integral between the limits -1 and $+1$ of the product of any three of the Coefficients. On the other hand, if we know the value of this definite integral, we can immediately deduce the development of the product of any two of the Coefficients. Thus it may be of interest to give a brief investigation of the value of the definite integral. I follow the notation adopted by Professor Adams.

The formula to be established is

$$\int_{-1}^{1} P_m P_n P_p d\mu = \frac{2}{2s+1} \frac{A(s-m)A(s-n)A(s-p)}{A(s)},$$

where $2s = m+n+p$, and the functional symbol $A(r)$ is thus defined: if r is a positive integer

Fig 3.